Application of Shell Jetting Analysis for Determination of the Virtual Origin in Shaped Charges

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ABSTRACT: Shaped charges are designed to produce high-velocity jets for penetration. During jet formation, the liner collapses and converges at a point source also known as the virtual origin (VO), along the distance-time plane. The location of the VO must be known to allow the development of penetration analytical models. Here we determined the VO position using the ANSYS® Autodyn 2D shaped charge jetting technique. Jetting analysis was conducted for two shaped charges of 18 and 32 mm diameter. The explosive and casing were represented by Eulerian two-dimensional finite difference grids whereas the liner was modelled using a shell formulation. The summary/history of the jetting analysis was used to determine the location of the VO of the shaped charges. Interpolating the point of intersection between the jet velocity (U_{Jet}) and the cumulative jet mass, on the liner, revealed the location of the VO at a distance equivalent to approximately two-thirds of the inner cone diameter of the shaped charges in agreement with earlier studies using

different methods. Validation of this technique using the DiPersio, Simon and Merendino (DSM) model based on the Allison-Vitalli equation also showed good agreement with the numerical results.

Key Words: Virtual origin, cumulative jet mass, jet velocity, jetting analysis, numerical simulation.

Introduction

Shaped charges are used extensively against armoured vehicles and structures. Enhancing the performance of the shaped charge by changing variables through experimental trials cost a lot, hence several analytical formulas were developed to predict the performance of shaped charges. DiPersio, and Simon's (1964) [1] presented explicit formulas which took into account the concept of virtual origin (VO)[2] proposed by Allison and Bryan(1957) [3] and developed by Allison and Vitalli (1963) [4] for continuous and particulated jets with non-uniform jet velocity distribution [5]. In particular, given target density (ρ_t) and jet density (ρ_j), jet tip velocity, (V_0) and cut-off velocity (V_c), DiPersio, and Simon [1] applied Equations 1-3, for calculating depth of penetration before, during, and after jet breakup [6][7].

for jet breaks before penetration;

$$P = Z_0 \left[\left(\frac{V_0}{V_c} \right)^{1/\gamma} - 1 \right] \text{ for } 0 \leq Z_0 < V_c t_b \left(\frac{V_c}{V_0} \right)^{1/\gamma} \quad 1$$

for jet breaks during penetration;

$$P = \frac{(1+\gamma)(V_0 t_b)^{\frac{1}{1+\gamma}} Z_0^{\frac{\gamma}{1+\gamma}} V_c t_b}{\gamma} - Z_0 \text{ for } V_c t_b \left(\frac{V_c}{V_0} \right)^{1/\gamma} \leq Z_0 < V_0 t_b \quad 2$$

for jet breaks before reaching the target;

$$P = \left(\frac{(V_0 - V_c)}{\gamma} \right) t_b \quad \text{for} \quad V_0 t_b \leq Z_0 < \infty$$

While these formulas gave reasonable agreement with experimental result [8], one of the limitation is the priori determination of 'Z_o' which is effectively the distance between the VO point and the target surface 'b' [9] also known as the effective jet length represented in Equation 4.

$$Z_o = VO - b$$

One of the established method for locating the VO is through back-projection of the jet particles by X-ray radiography [10]. Although accurate, the technique is very expensive and requires specialised equipment such as an X-ray source, coolers, capacitors and isolated fibre optics. Even so, such techniques have been applied [11] to determine the VO of shaped charges. For instance, the VO for large (150-mm calibre) shaped charge was located at a distance from the liner base equivalent to 64% of the charge diameter [12]. Several qualitative estimates have also been used for the computation of penetration depths, e.g. by assuming the VO position is located about one-third of the distance from the apex to the base of the cone [12] or by estimating the VO using past experience (rule of thumb') as three-quarters of the liner height [13]. Also, Held [14] found that the VO is located at a distance about 2/3 of the diameter inside the liner. In a recent study, the back projection technique was applied numerically using Euler solver in ANSYS® Autodyn 2D to back-project the locations of both the jet tip and the stagnation point at different times [11]. This technique requires the back-projection of the jet to be calculated at a defined time during the jet motion.

In this work, a simple Lagrangian coordinate technique postulated by Carleone and Chou (1974) [15] for relating the position of the jet element on the liner was developed to determine the VO of shaped charges. By obtaining the shaped charge parameters such as the cumulative jet mass and U-Jet (jet velocity) from a standard shell jetting analysis, the point where the jet velocity just overcomes the cumulative jet mass was marked the VO and this was related back to the exact position on the charge. A mesh sensitivity study is first described to determine the effect of mesh size and density of shaped charges before the technique is discussed on the 18 and 32 mm shaped charges.

Mesh sensitivity study

The application of ANSYS® Autodyn 2D to simulate shaped charges has been validated in several work, [16][17] particularly in representing jet velocity as a function of cumulative jet mass. Mesh sensitivity study was conducted for four uniform square mesh sizes: 0.2, 0.3, 0.5, and 0.7 to determine the effect of mesh size on jet tip velocity, penetration depth and the cumulative jet mass via a standard jetting analysis. A 32 mm shaped charge with 1.2 mm OFHC copper liner at 26.84° half-cone angle was used for the study. A C4 explosive charge was employed with a 2 mm thick aluminium 7039 casing. Material models for OFHC cu liner and aluminium casing were taken from the standard library of AUTODYN® while the inbuilt Jones-Wilkins-Lee EOS was applied for the C4 explosive.

Effect of mesh size on jet tip velocity

The explosive, liner and casing were modelled in Euler sub-grid and nodal monitoring points (velocity 'gauges') were placed at 2CD intervals as shown in Figure 1, to obtain the jet velocity.

Figure 1. Gauge points at 1, 2 and 3 cone diameters

The result of the simulations is detailed in Figure 2. It is immediately apparent that the coarse mesh size of 0.7×0.7 mm produced the lowest jet tip velocity of 4.11 km/s , compared to a velocity of 5.9 km/s produced by the smallest mesh size of 0.2×0.2 mm. A gradual decrease in the tip velocity was observed in all mesh sizes and this is in agreement with experimental result described in [2] of page 179. Notwithstanding, the finest mesh size (0.2×0.2) mm produced the most accurate result, the difference between the jet tip velocities of the 0.3 mm and the 0.2 mm mesh sizes is only 0.36 km/s (i.e. 5.9%). The 0.2 mm mesh size however requires nearly three times the time required to complete the simulation compared to the 0.3 mm mesh. Consequently, it was decided that, on balance computational cost, to move beyond a 0.3 mm mesh was too great given the diminishing returns and the 0.3 mm mesh size was selected for all simulation.

Figure 2. The Velocity-time histories for the jet at the gauge points for different mesh size.

Effect of mesh size on penetration depth

The jet formed from the Euler sub-grid was remapped for penetration into 1006 steel with a standard fixed mesh of 0.5 x 0.5 mm producing results that compared favourably with previous experiments [18]. The simulation was allowed to run until the jet was eroded in the target. The penetration depths for the different mesh sizes are summarised in Table 1. The smallest mesh size achieved the greatest penetration depth, reflecting the higher tip velocity recorded for finer mesh sizes.

Table 1. The effect of mesh size on penetration depth.

Mesh size (mm)	0.2	0.3	0.5	0.7
Penetration Depth (mm)	141	136	116	90
Time required (h)	32	13	5	2

The time required is based on a computer with an intel(R) core (TM) i7-6700HQ processor CPU at 2.6GHz 2592mhz.

A non-linear trend in mesh size and penetration depth was apparent, with increasing convergence as the mesh size was reduced. This is illustrated in Figure 3, which plots the key numerical data presented in Table 1.

Figure 3. Variation of penetration depth with mesh size.

Figure 3 clearly reveals a polynomial relationship between the calculated penetration depths and the mesh size. Extrapolating the corresponding best-fit curve to a theoretical mesh size of '0' mm provides a

corresponding penetration depth of 160.75 mm. This suggests that given a mesh size of 0.2 mm, the error in calculated shaped charge performance is only ~12%. A further reduction in mesh size would therefore increase the accuracy, but the advantage would be minimal. Table 1 indicates that more time is required to run simulations with smaller mesh sizes, as highlighted in Figure 4. Essentially, an additional 19 h was required to run the model using a 0.2 mm mesh rather than a 0.3mm mesh with only a 3.54% increase in accuracy (relative to the nominal '0' mm mesh size baseline depth of 160.75mm). The computational cost of simulations with mesh sizes below 0.3 mm mesh is therefore too great given the diminishing returns and a 0.3 mm mesh size was selected.

Figure 4. Penetration depth and computation time for various mesh sizes

Mesh sensitivity for standard jetting analysis

The shaped charge jetting analysis is based on continuum mechanics that is used to estimate the non-uniform collapse velocity, [19] which is used to calculate other jetting parameters based on PER theory [20]. In the jetting set-up, the explosive is modelled in the Euler sub-grid whereas the liner is modelled in the shell formulation with the generation of jetting points. A picture of the jetting points is shown in Figure 5.

Figure 5. Jetting Points on the Shell Copper Liner

Jetting calculations were conducted for four mesh sizes 0.2, 0.3, 0.5 and 0.7 mm. It was observed that the detonation wave travels along the liner surface with a decreasing collapse velocity from the apex to the base with more liner material forming the jet. A summary of the jetting analysis, in which the cumulative jet mass is plotted against the axial X-position for all four mesh sizes, is detailed in Figure 6. The relationship between the cumulative jet mass and the axial 'x' position (X-jet) for all mesh sizes was similar, bearing the same shape at the beginning, up to a coordinate distance of nominally 55 mm. Beyond this point, variations among the curve were observed with a general convergence of the solution observed towards the 70 mm position for a mesh size of 0.3 mm x 0.3 mm. The reasonable accuracy and computational time of the 0.3 mm square mesh makes it suitable for the simulation and was used for the entire jetting analysis.

Figure 6. Mesh sensitivity study for standard jetting analysis

Effect of variations of the number of nodes on the cumulative jet mass.

As this technique involves locating the VO using shell jetting analysis, it is necessary to investigate the effect of variation of the number of nodal points on the U-Jet and cumulative jet mass. Employing similar charge configuration, four jetting calculations were conducted with different number of nodes ($J=\max$): 10, 15, 20 and 25. The effect of node variations on the U-Jet and cumulative jet masses are plotted in Figure 7 and 8 respectively. From the jetting calculation, it is apparent that the maximum jet velocity remains the same on all 4 jetting calculation while the cumulative jet mass increases with increase in the number of nodal points. This is because more liner material is added in the calculation, something shown in Figure 8.

Figure 7. Relationship between jet velocity and cumulative jet mass on number of nodes.

Figure 8. Effect of number of nodes on the cumulative jet mass

Determination of the VO technique

This technique applies a single Lagrangian coordinate to relate the position of the jet element back to its original position on the liner. The technique was first developed by Chou and Flis [21] and applied by Carleone and Chou (1974) [15] to calculate the strain, jet length and jet radius. The axial liner position is described by the coordinate 'x' while the coordinate ' ξ ' describe the position of the jet given as:

$$\xi(x, t) = Z(x) + (t - t_o)V_j(x) \quad 4$$

Where t_o is the time the liner element first arrives at the axis, $Z(x)$ is the location of the formation, and $V_j(x)$ is the jet velocity which is a function of the liner position and time.

Figure 9 Relation between the liner coordinate x, and the jet coordinate [15].

From equation 4,

$$Z(x) = \xi(x, t) - (t - t_o)V_j(x) \quad 5$$

Applying the inverse velocity gradient based on the conservation of momentum, with respect to 'x' when 'x' is the VO,

$$V_j(x) = V_j(VO) = \frac{\int_0^{x_{VO}} V_j \left(\frac{dm_j}{dX} \right) dX}{\int_0^{x_{VO}} V_j \left(\frac{dm_j}{dX} \right) dX} \quad 6$$

By integrating the mass piled up along the jet velocity of the jet particles, a one dimensional jet is observed when the U-Jet just overcomes the cumulative mass effect created by the pilling up. The point that the jet begins to protrude from the collapse matrix is the VO and this is described in Figure 9 as the intercept point between the U-Jet and cumulative jet mass. An example of the technique is applied in an 18 and 32mm shaped charge with C4 explosive in a 2mm thick Al 7039 casing. Copper (OFHC) liners of constant thickness of 0.5 and 1.2mm were used for the 18 and 32mm shaped charges respectively, with a 26.5° half cone angle in both cases. Point initiation at the rear of the charge was assumed. The output jetting data, comprising U-Jet, cumulative jet mass and nodal points/jetting points, were then extracted and reduced for subsequent analysis.

Results and discussion

From the liner collapse data, it was found that the amount of liner mass entering the jet increases towards the base of the liner while the jet velocity (U-Jet) declines. The point at which the U-Jet intercepts the cumulative jet mass lies between the tenth and eleventh node on the 32mm charge. To obtain this position on the x-y plane, the jetting points were numbered in ascending order and the position (between tenth and eleventh node) was interpolated on the charge as shown in Figure 10.

Figure 10. Determination of VO for the 32 mm shaped charge

The VO was found to be at the 61.00mm mark which is 24.13 mm from the cone apex and 7 mm from the charge base. This position is 75.86% of the liner height (about three-quarters of the liner height from the conical apex and about one-quarter from the base), which is also two-thirds of the shaped charge liner base diameter. The technique was applied for the 18mm shaped charge and the VO appeared to flow in the same way. Our results were in agreement with the VO location obtained by Held, (1991) who identified the VO, to be about two-thirds of the inside diameter of the liner and also DiPersio et al, (1967) who determined the VO at a distance equivalent to about three-quarters of the liner height from the cone apex.

Validation of the technique

To confirm the validity of this technique, an experimental study is required however, this required the procurement of expensive equipment such as x-ray and its accessories described previously. The absence of these equipment necessitates the use of other techniques; one of which involves

investigating the effect of the number of nodes on the VO position. 4 additional jetting calculations was conducted, 5, 10 14 and 20 maximum nodes on the shell liner. It was observed that as the number of nodes increases issues associated with tangled polygons were observed and this were deleted from the simulations. Analysis of the results showed that when the maximum number of nodes is low, for instance, when J max= 5. The jet formation is distorted as shown in Figure 11, pushing the VO position towards the apex (the VO was 55% from the charge base). The effect of VO position on J Max is at appendix 1. A decrease in the proposed VO position was observed for an increase in the number of nodes. At higher J Max, the distance between the charge base and the VO decreases up to a point where further increase in the number of nodes gave no change in the VO position. At this point, the VO position is taken.

Figure 11. Distorted liner collapse resulting from low number of nodes (J Max = 5).

Validation of the VO Location

The technique was also validated by computing the penetration depth of the effective and traditional SOD using the DSM formula described in Equation 1-3. The jet tip velocity was obtained at 2CD with a nodal monitoring point/gauge as shown in Figure 12. By applying the jet tip correction, when the U-Jet is plotted against the cumulative jet mass, the modified jet velocity was obtained as 5.096 km/s.

Figure 12. Jet velocity recorded at 2 CD for 32mm shaped charge (without tip correction).

while the DOP was determined numerically using AUTODYN 2D. The explosive, liner and casing were modelled in the Euler sub-grid whereas the target was modelled with the Langregian sub-grid. The jet from the Euler solver was remapped to the Langregian solver for penetration into 1006 steel until the jet element was completely eroded. Figure 13 shows the DOP for the 32mm shaped charge.

Figure 13. Depth of penetration for 32 mm shaped Charges.

According to E. Hirsch [22], the penetration of an ideal jet into steel becomes highly ineffective below about 1.8km/s, when the rate of target penetration becomes less than half the jet velocity. However, deviations from perfect symmetry and jet particulation accounts for the loss in penetration efficiency below 3km/s. The cut-off velocities were determined from the penetration-time history (Figure 14) of the shaped charge jet described by M Held in [18] [23].

Figure 14. 32mm shaped charge Jet Penetration-time history.

Table 1. Parameters used to determine DOP.

V_{tip} (m/s) corrected	$\sim V_{(min)}$ (m/s)	SOD (mm)	Effective SOD (mm)	Solid jet density (g/cm ³)	Target density (g/cm ³)	$y = \sqrt{\rho_t / \rho_j}$
5096	1787	64.00	71.00	8.96	7.85	0.93

Having satisfied the condition $0 \leq Z_0 < V_c t_b \left(\frac{V_c}{V_0} \right)^{1/\gamma}$ for continuous jet, Equation 1 was applied. The DOP for the numerical simulations and analytical equations using the derived and traditional SOD are detailed in Table 3. It was observed that higher penetration depths were observed for the effective SOD compared with results of the numerical simulations. The proposed VO technique shows only a 4.5% difference in penetration depth from the numerical simulation while the traditional SOD under-predict the numerical simulation by 5.6%. The ability of this technique to determine the location of the VO as well the improved predictions obtained in comparison with the traditional SOD makes it valid for determination of the VO.

Table 2. Comparison of the depth of penetration of numerical simulations, SOD and derived SOD position.

Numerical Simulation at 0.3 mm mesh (mm)	Using the traditional SOD (mm)	Using the Effective SOD (mm)	Difference in DOP between traditional SOD and numerical simulation.	Difference in DOP between effective SOD and numerical simulation.	Difference in DOP between traditional SOD and Effective SOP
141.00	133.43	148.02	7.67~5.4%	6.92~4.5%	14.59mm

Conclusion

In conclusion, we carried out a series of numerical simulations to investigate the ability of ANSYS® Autodyn to predict the VO of shaped charges. The technique applies a simple Lagregian coordinate for relating the position of the jet element on the liner. It has been shown that:

- Ignoring the distance between the charge base and the VO underestimate the depth of penetration.
- The VO position can be reasonably estimated by Shaped charge jetting analysis using Ansys Autodyn 2D.
- The jet tip velocity remains the same irrespective of the number of nodes or J max on the shell jetting analysis.
- When the number of nodes is low, the jet formation is distorted therefore moving the VO towards the Apex.
- By computing the DOP using the derived VO, gives only 4.5% deviation from the numerical simulation while the traditional SOD gives 5.6% deviation.
- The proposed technique agrees well with earlier works (DiPersio et al., 1967; Held, 1988).

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