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Abstract

This research studies the post impact response of damaged area of variable stiffness curved composite plates. Varying thicknesses of sections is widely found in aerospace and automotive composite sub structures. In this regard, the impact response of this geometry characteristic has to be studied in thin-walled structures. In our model, a removal of ply technique is used to represent damaged region within a curved panel, thus degrading the stiffness in that area is considered in the theoretical models. A summation of spring-mass systems is used in the modelling of damaged variable stiffness plate to analysis post impact behaviour of these structures. The theoretical force-time results are also compared with the relevant finite element outcomes in LSDYNA. The comparison establishes a good prediction capability of the proposed model.

Keywords: Polymer-matrix composites (PMCs); Post impact; Variable stiffness; Damage

Nomenclature

$F(t)$
contact force of undamaged plate
 $F(t)_{damaged}$
contact force of damaged plate
 E
Young’s modulus (GPa)
 G_{12}
Shear modulus (GPa)
 γ
Poisson’s ratio of plate/impactor
 ρ
density of plate/impactor
 b
width of plate

M_1	
mass of variable stiffness plate	
M_2	
mass of impactor	
K_1	
stiffness constant of plate	
$K_{1(damaged)}$	
damaged stiffness constant of the plate	
K_2	
stiffness constant of striker	
K_2^*	
effective contact stiffness	
V	
impact velocity	
t_m	
thickness of variable stiffness section	
t	
time variable	
ω_{eq}	
equivalent natural frequency of variable stiffness plate	
$\omega_{eq(damaged)}$	
equivalent natural frequency of variable stiffness plate (damage induced)	

1 Introduction

Composite structures offer many advantages when compared to conventional materials such as high strength and stiffness to weight ratio and therefore the use of these materials have increased rapidly in the past decades for aerospace and automotive applications [1]. However, composites experience impact loads during their service life. Impact damage reduces the strength of laminated structures. There are several factors affecting the low velocity impact loading conditions of composites. Fibre type, resin type, lay-up, thickness, loading velocity and projectile type play a key role in impact dynamics of composite structures.[2].

Many researchers have investigated different mathematical models to predict the impact damage behaviour of various composite materials and structures. There is still a need to develop a mathematical model to predict the post impact response of variable stiffness curved plates. The main aim of this study is to develop a mathematical model to accurately predict the post impact response of damaged variable stiffness curved composite panels. The most recent models which are used to study the impact behaviour of composite structures are highlighted below.

Impact on composite structures has attracted the attention of many researchers worldwide. The elastic response of orthotropic laminated cylindrical shells was analysed by Gong et al. [3]. They proposed an

analytical force function based on material properties, mass of the shell/striker and the impact velocity to predict the impact force response. This force function was used depict contact-force histories for different cases of impactor masses and velocity.

Khalili et al. [4] studied the dynamic response of a thin smart curved composite panel subjected to a low-velocity transverse impact embedded with shape memory wires. Their work was based on the linear Hertzian contact model which is linearized for the impact analysis of the curved composite panel. The governing equations of the curved panel are provided by the first-order shear deformation theory and solved by Fourier series. Kistler and Waas [5] studied the influence of in-plane and out-of-plane boundary conditions, the effect of curvature and the validity of linear and non-linear plate theory on the transverse impact of composite cylindrical panels. They proved that as the thickness decreases, deformations increase and the effects of curvature becomes increasingly important. The study established the importance of considering bending and membrane effects for analysing impact on a curved plate and presented that these effects were more significant than inertia effects for the range of velocities and impact energies studied. Saghafi et al. [6] investigated the effects of preloading on the impact response of curved laminates. The upper and lower surfaces of the specimens were put under tensile and compressive stress respectively, and the panel curvature also increased. Their results showed that preloading the plate had a drastic effect on the impact parameters such as maximum displacement and damaged area. They proved that as the preload increases, the maximum load and displacement increased and decreased respectively. This was primarily due to the increase in stiffness of the laminate. Choi [7] numerically studied the transient response of composite laminated plate and cylindrical shells subjected to low velocity impact. Their results showed that plates/shells with large curvatures consistently exhibited smaller deflections and larger contact forces than the flat plate. Leylek et al. [8] developed a FE analyses on the low velocity impact on curved composite panels. It was shown that as the radius of curvature of the panel increases, the maximum contact force decreased. The mesh element ratio of impactor and composite panel played an important role and they demonstrated that the FE analyses could be used efficiently in the impact response of curved fibre composites.

Goo and Kim [9] developed a three dimensional finite element analyses to simulate the dynamic behaviour of composite laminates. They pointed out limitations of the modified Hertz contact law such as its inability to account for span or thickness and stacking sequence. They investigated the impact force histories of curved composite laminates with various curvatures and stacking sequences and discussed the effects of curvature on the impact behaviour of composite laminates.

Shivakumar et al. [10] applied spring-mass and energy balance models to predict the impact force on circular graphite-epoxy laminates. In their model a two-degree of freedom model is performed consisting of four springs for bending, shear, membrane and contact deformation characteristics. However, the impact force was calculated without considering the damage effects of the plate. Studies carried out by Singh and Mahajan [11] depicted that the force and deflection response of the laminate is largely dependent upon the extent of damage in composite laminates. Their model was able to predict the inter-laminar and intra-laminar damage effects on the stiffness of the structure. The damage caused a degradation of stiffness at the impact location and therefore lower contact force. The FE simulations proved that damage changes the nature of the impact force time history. Matrix cracking and delamination cause significant change in the characteristics of the plate which are dependent on the extent of damage. These issues were addressed by Olsson [12-14] and he developed an analytical model to predict small mass impact on plates with delamination growth and damage. Shahid et al. [15] recognized the impact force response as a major parameter characterising impact damage resistance of laminated composites. They developed an analytical model to predict the damage resistance of composite laminates beyond the initial damage state through modifying the spring constant in Hertzian contact law which is a function of the extent of damage in composites.

Arachchige and Ghasemnejad [16] recently developed a theoretical model to predict the transverse impact of curved variable stiffness plates. According to their results, changing the thickness of sections vary the impulse response of composite panels. The model was based on the first order shear deformation theory and response of variable stiffness composite plates are predicted with a range of layups and geometry design under low velocity impact loading conditions.

The present model developed a summation of spring-mass model to predict the contact force between a striker and a damaged curved plate with variable stiffness during the impact. The analytical model presented here includes the effect of damage on the overall stiffness of the laminate and also on the force-time response. The force-time histories are compared with the FE simulations and a good correlation was established.

2 Analytical impact modelling

First order shear deformation theory coupled with Double Fourier series has been used to solve the dynamic contact problem which is derived through the expansion of loads, displacement and rotation functions. All details of the principle calculations can be found in previous work of authors [3]. Fig 1 shows a schematic diagram of impact on a variable stiffness plate. In this plate sections 1, 2 and 3 corresponds to variable stiffness sections with different thicknesses Fig. 2.

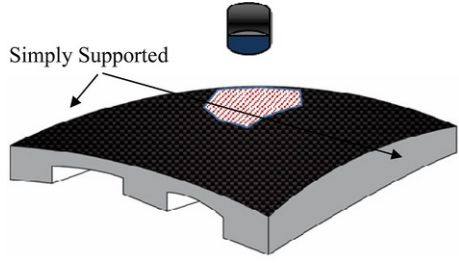


Fig. 1 Schematic diagram of damaged curved composite plate with variable thickness (stiffness).

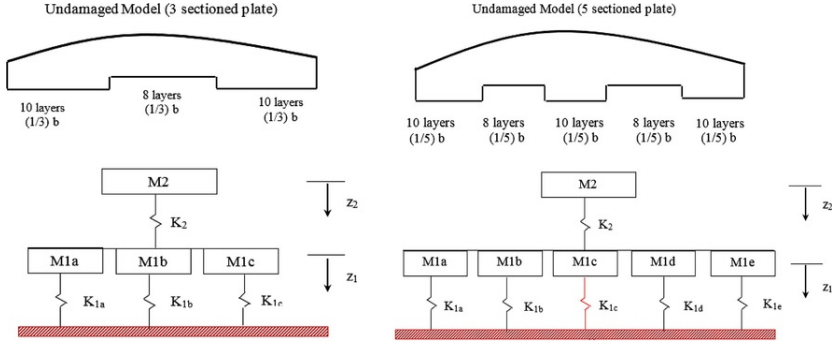


Fig. 2 Mass-Spring diagram of (a) three section plate (b) five section plate.

Love's equations of motion for a curved shell of dimensions a and b , radius R and thickness h under external loads [4] are expressed as:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} + q_x(x, \theta, t) = \rho h \ddot{u} \quad (1)$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_\theta}{\partial \theta} + \frac{Q_\theta}{R} + q_\theta(x, \theta, t) = \rho h \ddot{v} \quad (2)$$

$$\frac{\partial Q_x}{\partial x} + \frac{1}{R} \frac{\partial Q_\theta}{\partial \theta} - \frac{N_\theta}{R} + q_n(x, \theta, t) = \rho h \ddot{w} \quad (3)$$

$$\frac{\partial M_x}{\partial x} + \frac{1}{R} \frac{\partial M_{x\theta}}{\partial \theta} - Q_x = \frac{\rho h^3}{12} \ddot{\beta}_x \quad (4)$$

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial M_\theta}{\partial \theta} - Q_\theta = \frac{\rho h^3}{12} \ddot{\beta}_\theta \quad (5)$$

In this paper, the spring-mass model proposed by Gong et al. [3] for cylindrical composite shells is adopted. The contact force between the impactor and the laminated shell during the impact event is governed through the Hertzian theory which is defined by,

$$F_c(t) = K_2 \delta^p \quad (6)$$

The force function derived by Gong et al. [3] is defined by;

$$F_c^*(t) = K_2^* [A_1(C_1 - 1) \sin \omega_1 t + A_2(C_2 - 1) \sin \omega_2 t] \quad (7)$$

This approach uses an effective contact stiffness, K_2^* which results in obtaining an analytical solution for the impact force since Eq. (6) possesses a risk due to its high nonlinearity

$$K_2^* = \sqrt{\pi} \Gamma \left(\frac{p+1}{2} \right) \frac{2\Gamma \left(\frac{p}{2} + 1 \right) + \sqrt{\pi} \Gamma \left(\frac{p+1}{2} \right)}{4\Gamma^2 \left(\frac{p}{2} + 1 \right) + \pi \Gamma^2 \left(\frac{p+1}{2} \right)} \delta_m^{p-1} K_2 \quad (8)$$

The constants in the force function in equation [7] are defined by

$$\omega_{1,2}^2 = \frac{1}{2} \left(\frac{k_1 + k_2}{M_1} + \frac{k_2}{M_2} \right) \mp \sqrt{\frac{1}{4} \left(\frac{k_1 + k_2}{M_1} + \frac{k_2}{M_2} \right)^2 - \frac{k_1 k_2}{M_1 M_2}}$$

(9)

$$C_1 = \frac{k_2}{k_2 - \omega_1^2 M_2}$$

(10)

$$C_2 = \frac{k_2}{k_2 - \omega_2^2 M_2}$$

(11)

$$A_1 = \frac{V}{\omega_1(C_1 - C_2)}$$

(12)

$$A_1 = \frac{V}{\omega_2(C_2 - C_1)}$$

(13)

3 Analytical modelling of a variable stiffness curved plate (undamaged plate)

In this section, the analytical model developed by Arachchige et al. [16] is adapted to model firstly a curved composite plate with three variable stiffnesses and secondly the one with five sections. Damage models were also developed for these two models. The material properties of the plate and the impactor are given in Tables 1 and 2.

Table 1 Properties of steel impactor [3].

Young’s Modulus	207 GPa
Poisson’s ratio	0.29
Density	7900 kg/m³
Diameter	20 mm
Mass of the striker	3 kg
Velocity of striker	6 m/s

Table 2 Properties of CFRP composites [3].

E ₁	141 GPa
E ₂ = E ₃	9.7 GPa
G ₁₂ = G ₁₃	5.5 GPa
G ₂₃	3.7 GPa
ν ₁₂ = ν ₁₃ = ν ₂₃	0.29
Density	1530 kg/m³

The equations for the natural frequency, ω_{eq} derived in [4,16] to incorporate variable stiffness sections and derived as:

$$\omega_{eq} = \int_0^b \sqrt{\sum_{n=1}^m \frac{-(C_{13}K_U + C_{23}K_V + C_{33} + C_{34}K_X + C_{35}K_Y)_m}{\rho t_m}}$$

(14)

where m is the number of variable stiffness sections. Eq. (14) is developed to include the position of the variable stiffness section. The analysis is performed by integrating through the width of the plate and t_m

is the thickness of variable stiffness section. Definition for all constants in Eq. (14) are derived in [4].

Removal of plies is equivalent to the reduction in the total thickness. The damaged plates are repeatedly subjected to impact to quantify the effect of the damage on the contact force. Each time the thickness of the damaged region is reduced, so the [ABD] stiffness matrices for the whole plate are re-calculated and it is observed that the natural frequency of the damaged plate governed by Eq. (14) reduces, thus it causes degradation of stiffness.

4 Analytical modelling of a variable stiffness curved plate (damaged plate)

This section of the paper focusses on the damage modelling of the variable stiffness models developed in Ref. [3] and modified in section 4. A new approach for damage modelling is derived through the reduction of thickness in the damaged region, subsequently reducing the natural frequency and stiffness in that region subjected to damage. The reduced stiffness is achieved through reducing the damaged ply thickness to zero. The damage model is split into seven sections for the 3 sectioned plate and nine sections for the 5 sectioned plate with reduced stiffness. The thickness of the variable stiffness section subjected to impact is decreased in order to simulate the damaged layers. The validity of this approach is verified through the finite element analysis carried in latter sections. The overall natural frequency can be calculated in Eq. (14). This natural frequency is for the plate with three sections. Due to the introduction of the damaged region, there will be seven variable stiffness regions. The natural frequencies for the sections of the plate are calculated considering the location of the variable stiffness section. The damaged section is defined by the dimensions of the impactor and corresponds to an area $20 \text{ mm} \times 20 \text{ mm}$, thus leads to refine the proposed model. This damage model assumes that the damage caused by the spherical impactor which is equivalent to a square region of $20 \text{ mm} \times 20 \text{ mm}$ (see Fig. 3).

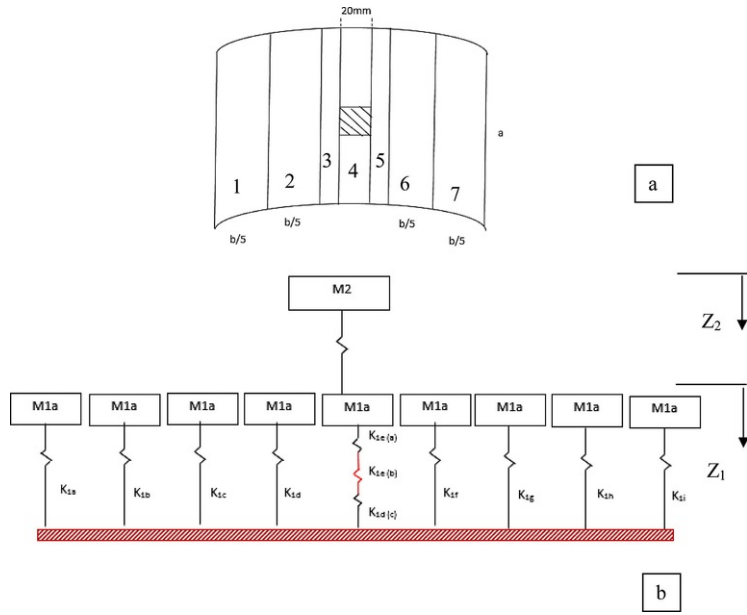


Fig. 3 Two-degrees-of-freedom spring mass model of damaged 5 sectioned variable thickness plate.

The equivalent natural frequency of Section 4 $\omega_{eq(4)}$ is defined as:

$$\omega_{eq(4)} = \frac{1}{\omega_{eq(4a)}} + \frac{1}{\omega_{eq(4b)}} + \frac{1}{\omega_{eq(4c)}} \quad (15)$$

K_{1d} represents the equivalent stiffness of region 4 which consists of the damage region. The damage region is modelled as an area of $20 \text{ mm} \times 20 \text{ mm}$ where the diameter of the impactor is 20 mm . The expression of the equivalent stiffness of region 4 is expressed as:

$$K_{1d} = \frac{1}{K_{1d}a} + \frac{1}{K_{1d}b} + \frac{1}{K_{1d}c} \quad (16)$$

In this model the stiffness constants K_{1da} and K_{1dc} represents stiffnesses of undamaged regions in Section 4. Therefore, in this model the damage region is refined.

5 Results and discussions

The layups analysed in the study are listed in Table 3. The same layups are used as in Ref. [16] to maintain linearity in comparison. In each case the no. of sections and the layups of the undamaged and damaged model are displayed. The layup where the impact damage occurs is highlighted in red in the damage model. In all cases, the orthotropic nature of the laminate is maintained since the above derived equations are valid only for an orthotropic composite laminate (see Table 3). The layups for the undamaged model are included in the previous work of authors [16]. The ply configuration of the damaged region is highlighted.

Table 3 Various layups for the variable curved composite panels [3].			
Layup	No. of sections (Undamaged Model)	No. of Sections (Damaged Model)	Layup (Damaged Model)
1	3	7	[−45/45/0/90/0] _s [−45/45/0/90] _s [−45/45/0/90] _s [−45/45/0/90]_s [−45/45/0/90] _s [−45/45/0/90] _s [−45/45/0/90/0] _s
2	3	7	[0/90/0/45/−45] _s [0/90/−45/45] _s [0/90/−45/−45] _s [0/90/−45/−45]_s [0/90/−45/−45] _s [0/90/−45/45] _s [0/90/0/45/−45] _s
3	3	7	[0/90/0/90/0] _s [0/90/0/90] _s [0/90/0/90] _s [0/90/0/90]_s [0/90/0/90] _s [0/90/0/90] _s [0/90/0/90/0] _s
4	5	9	[−45/45/0/90/0] _s [−45/45/0/90] _s [−45/45/0/90/0] _s [−45/45/0/90/0] _s [−45/45/0/90/0]_s [−45/45/0/90/0] _s [−45/45/0/90/0] _s [−45/45/0/90] _s [−45/45/0/90/0] _s
5	5	9	[0/90/0/45/45] _s [0/90/−45/45] _s [0/90/0/45/−45] _s [0/90/0/45/−45] _s [0/90/0/45/−45]_s [0/90/0/45/−45] _s [0/90/0/45/−45] _s [0/90/−45/45] _s

			[0/90/0/45/−45] _s
6	5	9	[0/90/0/90/0] _s [0/90/0/90] _s [0/90/0/90/0] _s [0/90/0/90/0] _s [0/90/0/90/0]_s [0/90/0/90/0] _s [0/90/0/90/0] _s [0/90/0/90] _s [0/90/0/90/0] _s

This paper studies the effect of damage on the contact force histories of the plate with 3 sections and 5 sections respectively for the layouts in Table 3. A ply removal technique is adopted which reduces the thickness of the damage region and thus the stiffness is degraded in that region. Through this analysis, the best layouts that can withstand impact damage for variable stiffness composites can be deduced. The present analysis proves that Layout 1 ([−45/45/0/90 0]_s/[−45/45/0/90]_s/[−45/45/0/90/0]_s) yielded the highest impact force without damage. The maximum contact force without damage was 2% higher than layout 2 and 28% higher than layout 3. The results showed that inclusion of more 0° plies resulted in a relatively lower maximum impact force. For the damaged model, the 3 sectioned plate was split into 7 sections to incorporate damage where the damaged region was modelled as a 20 mm × 20 mm square. Removal of plies were done through this section. The removal of a single ply from the damaged region resulted in a maximum impact force drop of 6% of the undamaged plate for Layout 1. For Layout 2, this drop was 16% and for Layout 3 was 17%. The highest drop in impact force for Layout 1 was noticed in between removing plies 5 and 6. This drop was 48% when compared to 41% of Layout 2's highest drop of force observed between removing plies 6 and 7. The highest difference between impact forces for Layout 3 which is 44% was observed in between ply removal 7 and 7. This analysis shows that significant weakening of the structure was observed between ply removals 5 and 7. The analysis also proved that Layout 2 was the best design for the plate with 3 sections, since the induced damage affected the impact force history lesser and mainly since it had a higher contact force after removing all plies from the damaged region (see Fig. 4).

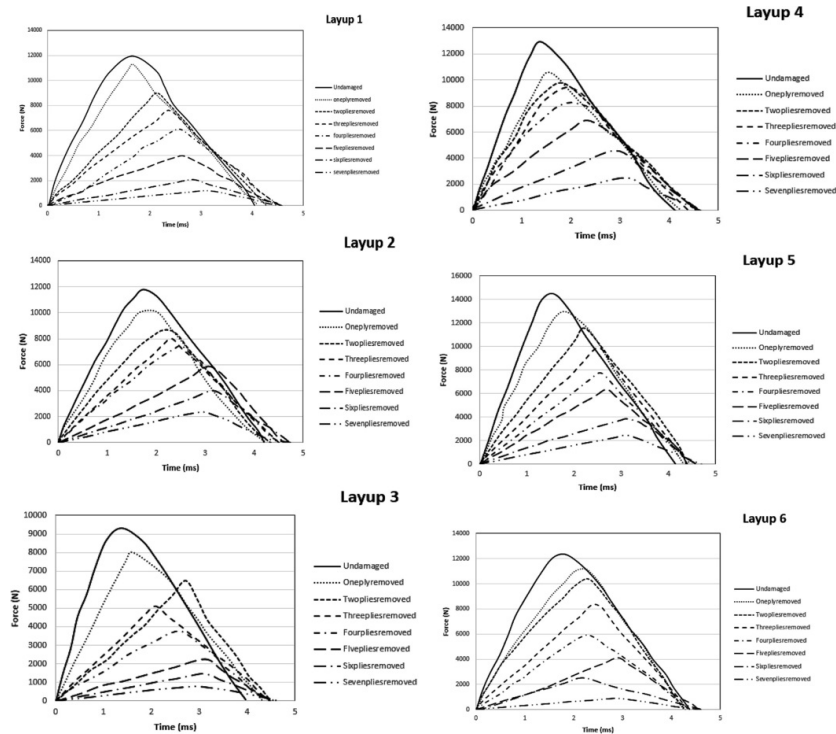


Fig. 4 Comparison of force – time history of undamaged and damaged models for various layouts.

The Layups 4, 5 and 6 are associated with the damage plate with 5 sections. The present analysis proved that Layup 5 [0/90/0/45/−45]_s/[0/90/−45/45]_s/[0/90/0/45/−45]_s/[0/90/−45/45]_s/[0/90/0/45/−45]_s yielded the highest contact force of 14,488 N. This maximum force was 12% higher than that of Layup 4 and 17% higher than that of Layup 6. It is observed that shifting the position of the 45° and −45° plies in the laminate played an integral role in the change of the impact force. The 5 section plate was split into 9 sections to incorporate damage as shown in Fig. 4. The removal of a single ply from the damaged region resulted in an impact force drop of 18% of the undamaged plate for Layup 4. For Layup 5, this drop was 10% and 9% for Layup 6. Therefore, a higher impact force drop was observed at damage initiation in Layup 5. The highest drop in impact force for Layup 4 was noticed in between removing plies 6 and 7. This force drop was 46% when compared to 65% of Layup 5’s highest force drop observed between removing plies 5 and 6. The highest difference between impact forces for Layup 6 which is 64% was observed in between ply removal 6 and 7. Layup 4 was able to sustain the highest impact force after removal of all plies proving that it is the best layup design able to withstand damage even when Layup 5 yielded a higher impact force without damage induced (see Fig. 4).

5.1 Explanation of damage behaviour

It can be observed from Fig. 4 that damage causes a degradation in the overall stiffness of the plate. The damage was introduced into the model through the reduction in the thickness of the damaged section in the plate, thus degrading the overall natural frequencies of the plate. Therefore, the maximum contact force decreases as observed in Fig. 4. The slope of the graphs in Fig. 4 decreases as the thickness of the damaged region is reduced. This implies that the stiffness of the plate is decreasing, thereby weakening the laminate. However, contact duration is increasing as more damage is introduced. The first load drop, in terms of Hertzian failure or significant damage corresponds to occurrence of initial damage in the form of matrix cracks or delamination as stated in literature. Damage initiation in the present model is based on removal of the first ply of the damaged region. In the present model, the total contact duration increases as the level of damage increases as shown in Table 4.

Table 4 Contact Time duration for the different layups.

Contact time duration (ms)							
Layups	1ply removed	2plies removed	3plies removed	4plies removed	5plies removed	6plies removed	7plies removed
1	4.1	4.3	4.5	4.6	4.6	4.6	4.6
2	4.2	4.3	4.5	4.5	4.6	4.7	4.7
3	4.3	4.5	4.5	4.5	4.6	4.6	4.6
4	4.2	4.4	4.5	4.6	4.6	4.7	4.7
5	4.3	4.4	4.4	4.5	4.6	4.6	4.7
6	4.3	4.3	4.4	4.4	4.6	4.6	4.7

5.2 Effect of curvature on damage behaviour

The effect of curvature of the plate on the damage behaviour is studied analytically. It is observed that as the curvature of the plate increases, the contact force is reduced. In the case of the undamaged plate, a drop in the maximum contact force of 9% is observed when the radius of the plate increases from 0.15 to 0.20 m. As damage is initiated through removal of the first ply, the force drop is 15% for the same increase in radius of the plate. This proves that the radius of the plate has an effect on the damage characteristics of the plate (see Fig. 5).

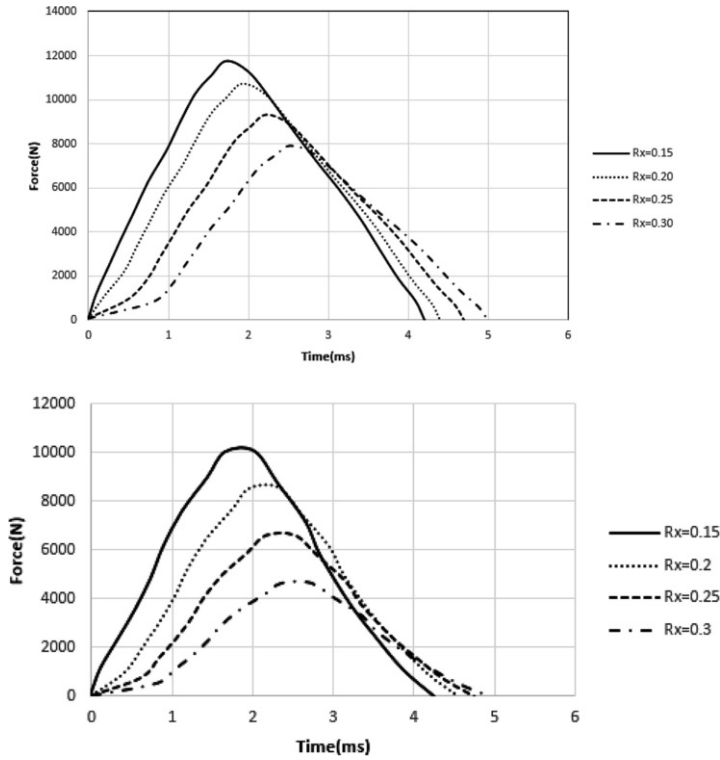


Fig. 5 Effect of radius of the plate on the progression of damage (radiuses are in meter).

6 Validation of theoretical model using FEM

The damaged curved plates were modelled using finite element software LSDYNA. The size of the composite plates was $600 \times 200 \text{ mm}^2$ and radius 150 mm with variable thickness through the length of plate. The plates were modelled based on Belytschko-Lin-Tsay quadrilateral shell elements. This shell element is based on a combined co-rotational and velocity strain. All surfaces of the model were meshed using quadratic shell element and the size of an element was $1 \times 1 \text{ mm}^2$ in the middle of beam. The impactor was modelled as a rigid block using solid element. MAT_54 was used as the material model for the plate with Chang-Chang failure criterion. Material model MAT_54 in LS-DYNA is commonly used to simulate composite failure. This model categorizes four failure modes: Tensile fibre failure mode, compressive fibre failure mode, tensile matrix failure and compressive matrix failure modes. These failure indicators are appointed on total failure for the lamina, where both the strength and the stiffness are set equal to zero after failure is encountered. The layers of each section was also defined using integration point (IP) through the thickness of the element and each integration point is used to represent each layer. In this case, the thickness of integration point layers at those places which are allocated for damaged zone was reduced to zero. This strategy introduces the damaged zone at the particular location. The mass of the impactor was 3 kg with an initial velocity of 6 m/s (Fig. 6).

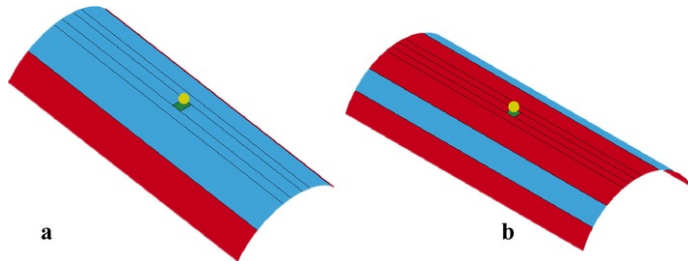


Fig. 6 Finite element models of damaged curved composite plates a) three sectioned and b) five sectioned.

In order to validate the theoretical model, a numerical finite element was developed and the results are analysed and compared in this section. The analysis is validated using two different layups for the two plates with different geometries since the numerical modelling process is time consuming to be done for all the combinations discussed in the previous section. The 3 sectioned variable stiffness plate is validated through using Layup 3 and 5 sectioned plate is validated using Layup 4. In [Figs. 7 and 8](#) the force-time history of damaged panels which were extracted from FEA model is presented. The main reason for difference between FEA and experimental results might come from deletion of elements after failure of all composite layers during the impact simulation. In this case there is no more resistance against the striker, therefore, some discrepancies are observed between experimental and FEA results (see [Fig. 9](#)).

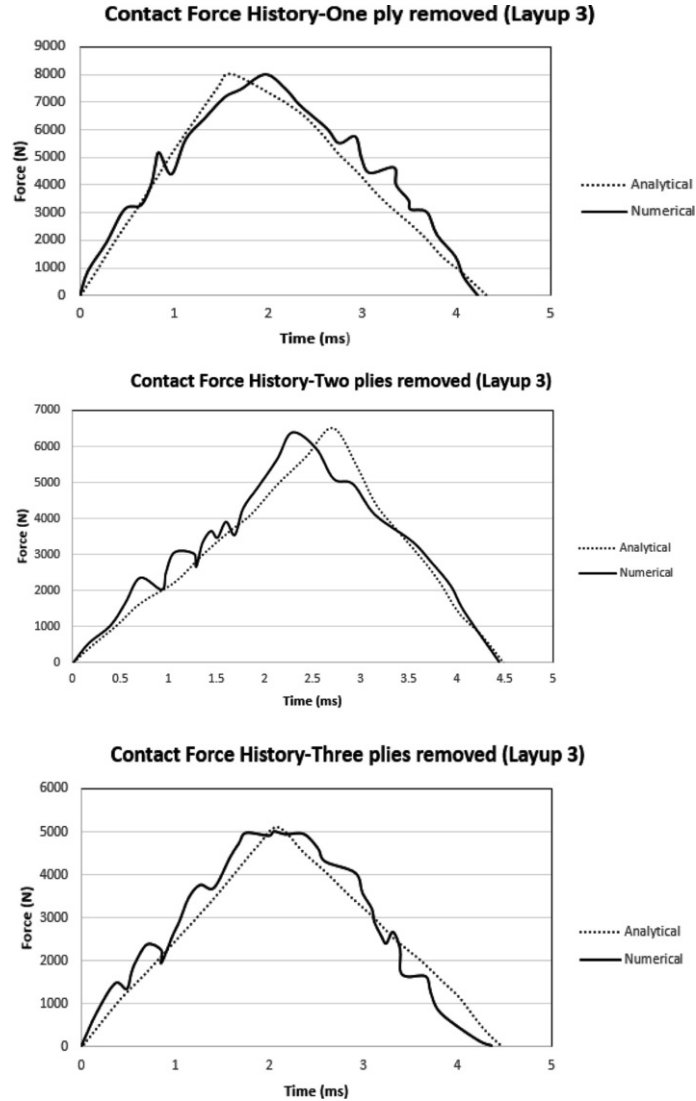


Fig. 7 Comparison between FE results and the developed theoretical model for layup 3.

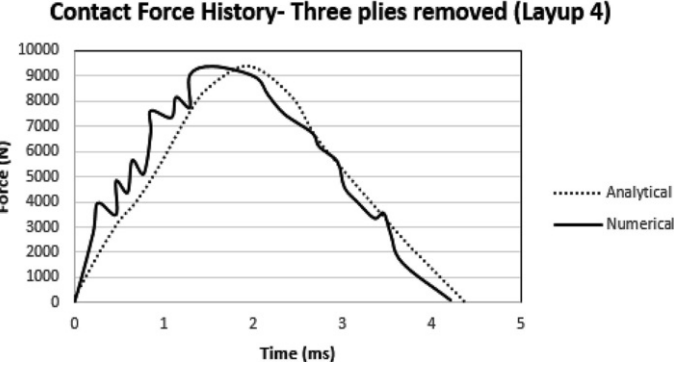
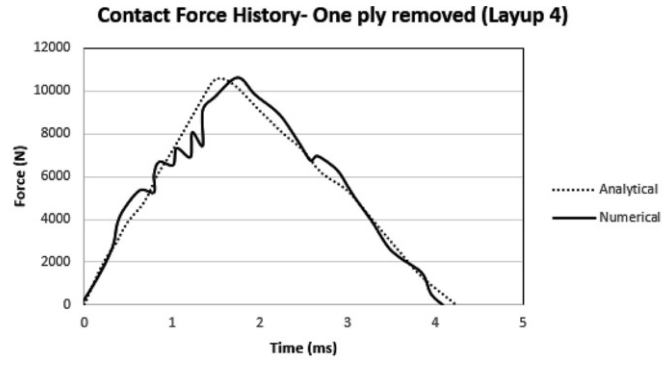


Fig. 8 Comparison between FE results and the developed theoretical model for layup 4.

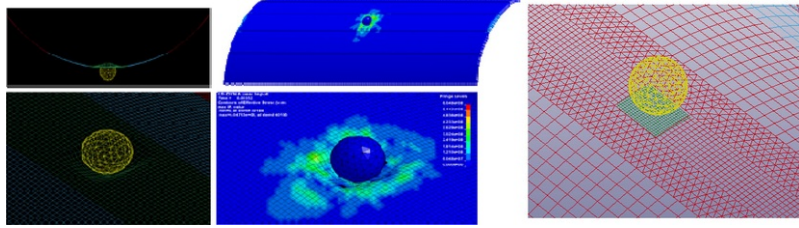


Fig. 9 Finite element images of damaged curved composite panels with variable stiffness a) element deformation, b) stress distribution and c) mesh generation.

7 Conclusion

This paper studies the post impact behaviour of variable stiffness curved composite plates. Variable stiffness sections are used widely and the need for studying the impact behaviour of these sections were identified. The existing damage models have not been developed to predict the impact damage behaviour of variable stiffness sections. In this regard, this paper adopts a new methodology to model the damage in the impacted region. The methodology of reducing the thickness of the damage region by ply removal was observed to be successful. The ply removal simulates weakening of the structure by degrading the stiffness in the impact region. Thus, the contact force is reduced when damage is introduced into the system. This research also investigated various types of layups able to withstand a greater impact damage mainly by focussing on the contact force at impact. Therefore, our research gives an insight into the combinations of layups most suitable for designs manufactured from the technique of variable stiffnesses in composites.

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