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ON THE PERFORMANCE SIMULATION OF INTER-STAGE TURBINE REHEAT

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sk-koehler@web.de**ABSTRACT**

Several authors have suggested the implementation of reheat in high By-Pass Ratio (BPR) aero engines, to improve engine performance. In contrast to military afterburning, civil aero engines would aim at reducing Specific Fuel Consumption (SFC) by introducing 'Inter-stage Turbine Reheat' (ITR). To maximise benefits, the second combustor should be placed at an early stage of the expansion process, e.g. between the first and second High-Pressure Turbine (HPT) stages.

The aforementioned cycle design requires the accurate simulation of two or more turbine stages on the same shaft. The Design Point (DP) performance can be easily evaluated by defining a Turbine Work Split (TWS) ratio between the turbine stages. However, the performance simulation of Off-Design (OD) operating points requires the calculation of the TWS parameter for every OD step, by taking into account the thermodynamic behaviour of each turbine stage, represented by their respective maps.

No analytical solution of the aforementioned problem is currently available in the public domain. This paper presents an analytical methodology by which ITR can be simulated at DP and OD. Results show excellent agreement with a commercial, closed-source performance code; discrepancies range from 0% to 3.48%, and are ascribed to the different gas models implemented in the codes.

KEYWORDS

gas turbine, simulation, inter-turbine, inter-stage, reheat, performance modelling, aircraft engines, industrial engine

NOMENCLATURE

BPR	By-Pass Ratio	[-]
CPR	Compressor Pressure Ratio	[-]
c_p	heat capacity at constant pressure	[J/(kg*K)]
DP	Design Point	
ITR	Inter-Turbine Reheat / Inter-stage Turbine Reheat	
HP	High Pressure	
HPT	High-Pressure Turbine	
HPT1	1 st stage HPT	
HPT2	2 nd stage HPT	
N	shaft physical rotational speed	[rad/s]
NDMF	Non-Dimensional Mass Flow	[(kg/s)*(K ^{0.5})/(Pa)] or [kg/s]
NGVs	Nozzle Guide Vanes	
OD	Off-Design	
P	Total pressure	[Pa]
PCN	compressor relative rotational speed	[-]
PLI	Power Law Index	[-]
PR	Pressure Ratio (also called Expansion Ratio in turbines)	[-]
SBTR	Second Burner Temperature Ratio	[-]
SFC	Specific Fuel Consumption	[(kg/s)/MN]
T	Total temperature	[K]
TET	Turbine Entry Temperature	[K]
TET1	1 st burner TET	[K]
TET2	2 nd burner TET	[K]
TW	Turbine Work	[W]

TWS	Turbine Work Split	[-]
W	mass flow	[kg/s]
γ	heat capacity ratio	[-]
η	efficiency (for other than turbomachinery)	[-]
η	isentropic efficiency (for turbomachinery)	[-]
η_p	polytropic efficiency	[-]
π_{burn}	pressure losses fraction through the burner	[-]
π_{intake}	total pressure recovery coefficient for the engine intake	[-]
in	(subscript) inlet of the component	
out	(subscript) outlet of the component	
in1	(subscript) inlet of HPT1	
out1	(subscript) outlet of HPT1	
in2	(subscript) inlet of HPT2	
out2	(subscript) outlet of HPT2	
1	(subscript) related to HPT1	
2	(subscript) related to HPT2	
DP	(subscript) related to DP	
OD	(subscript) related to OD	

1. INTRODUCTION

Attempts to further increase the efficiency and power output of industrial gas turbines and aero engines resulted in the study of various cycle extensions. Amongst them, reheat consists in a second heat addition in the main gas path, downstream of the main combustor. While this concept is widely used as ‘afterburning’ for short-period thrust augmentation in military low-bypass turbofans and turbojets, many authors suggest its application in large civil aero engines, for size reduction and efficiency improvement. Sirignano and Liu [1], and El-Maksoud [2] propose a continuous combustion throughout the turbine for simultaneous expansion and heat addition, resulting in near-constant temperature combustion. A concept with more practical relevance considers one or multiple discrete reheats along the expansion section and this has been studied by Vogeler [3], Chen et. al. [4] and Bergantzel and Waters [5]. Their results show a potential for significant performance improvements, when introducing a second burner. Apart from performance benefits, Lindvall and Conzelmann [6] discuss further potential advantages of ITR in industrial gas turbines, namely operational flexibility and reduced emissions.

In order to increase thermal efficiency in power generation (typically single-shaft) or aerospace propulsion (typically 2-shaft or 3-shaft) applications, the second combustor should be positioned very early in the expansion process [7]. In case of a HP spool with a 2-stage HPT, the reheat must take place between the first and second stage of the HPT to allow for heat addition at high pressure. However, the above-mentioned papers either include only DP performance studies or refer solely to engine configurations with one burner installed between turbines on different shafts. The current research work extends the known practices to OD simulation of engines with inter-stage reheat.

For detailed performance assessments of ITR and cycle optimisation studies, the required methodology should be able to model engines with multiple turbine sections on the same shaft. Moreover, to investigate operability issues and potential part-load benefits, it also needs to be capable of OD calculations, where the work split between the separate stages changes according to the thermodynamics and component characteristics. GasTurb [8], which is a commercial gas turbine simulation program, features ITR, but its application is limited to single-shaft industrial gas turbines with a pre-built setup of bleeds and cooling flows. Therefore, it is not applicable to studies of aero applications or gas turbines with sophisticated secondary cooling flows and several spools.

This paper describes a methodology to simulate multiple turbine stages on the same shaft. The methodology was developed with regard to its application in ITR engine studies, however, the detailed modelling of separate turbine stages also provides advantages in the accurate representation of turbine cooling flows, which is of particular importance to industrial gas turbines with multiple turbine stages on a single spool.

The aforementioned methodology is implemented into TURBOMATCH (the Cranfield University in-house 0-D performance simulation code, already featuring OD and transient calculations). TURBOMATCH [9] [10] is a gas turbine performance simulation tool, where the user defines the engine’s architecture through the declaration of the sequence of turbomachinery components (“bricks”) and the specification of their respective properties. This modularity allows for high flexibility and for a theoretically unlimited number of bleeds and cooling flows (i.e. it allows for the sophisticated modelling of both the main gas path and secondary air system). Therefore, it is particularly useful for research purposes, e.g. the study of complex flow arrangements and accurate representation of real engines that have multiple compressor bleeds for turbines’ nozzle guide vanes (NGVs) and blade cooling. The code has been successfully validated against experimental, test and simulated data [11] [12] [13].

2. THEORY / CALCULATIONS

2.1. DP TURBINE THERMODYNAMICS

This section explains the use of concatenated, simplified turbine characteristics for the determination of the DP operating point. The expansion section of a simple turbojet engine with ITR is considered, consisting of two consecutive turbine stages, followed by a convergent exhaust nozzle. This setup is shown in Figure 1.

Figure 1 - Turbojet engine with ITR

First, by looking at Figure 1 it can be observed that a crucial parameter in the design of the expansion section is the TWS. This parameter determines the ratio by which the interconnected turbine stages share the overall work (i.e. the work required by the compressor, plus any auxiliary/output work) between themselves.

$$TWS = \frac{\text{Stage 1 turbine work}}{\text{Overall turbine work}}$$

Equation 1

Equation 1 is sufficient to determine the work split in a setup with two turbine sections on one shaft. In configurations with more than two turbine sections, an individual TWS can be defined for each stage as follows:

$$TWS_{\text{stage } i} = \frac{\text{Stage } i \text{ turbine work}}{\text{Overall turbine work}}$$

Equation 2

For simplicity, the following discussions focus solely on configurations with two turbine sections on one shaft.

In cycles with inter-stage combustion, the work split determines the pressure at which the gas is reheated. By definition, a low TWS, i.e. a low expansion ratio in the first stage, enables heat addition at a higher pressure, thus lower entropy generation and higher thermal efficiency. Conversely, greater initial expansion before the reheat yields a higher potential for core specific power augmentation. More detailed considerations can be carried out by assuming the Turbine Entry Temperature (TET) to be equal at the inlet of the two turbine sections (this is a common assumption in reheated cycle studies, where the TET is usually equal to the maximum value allowed by the technological limitations). Under this assumption, the raising trend of the core specific power with the expansion ratio upstream the reheat combustor stops when the expansion ratios upstream and downstream the reheat burner become equal. If the expansion ratio upstream of the reheat combustor is increased even further, the core specific power decreases [14]. The value of the TWS must be specified by the user during performance modelling and is related to the design of the turbine blade rows and desired stage loading.

The aforementioned considerations explain why the DP work distribution between the HPT stages must be carefully selected during the design phase of any ITR cycle. The TWS is defined for DP operating conditions according to the turbine blade row design, but its value varies during OD operation according to the turbine performance characteristics, as explained in the following section.

In order to determine the theoretical DP operating conditions by means of concatenated turbine maps, the following assumptions can be made without limiting the theoretical validity of the methodology:

- pressure losses between components are neglected;
- pressure losses in 2nd burner are neglected;
- cooling flow injections in hot section are neglected;
- fuel flow in the 2nd burner negligible, compared to the main mass flow;
- both turbines and nozzle are choked at DP;
- turbines are NGVs-choked, therefore single-line turbine characteristics can be used.

In Figure 2 and Figure 3, the turbines are represented by single-curve characteristics, depicting the NDMF, both at the inlet (solid lines) and outlet (dashed lines) of the stage, against the total expansion Pressure Ratio (PR). This is the typical method for the representation of OD turbine performance [14] [15]:

$$NDMF = \frac{W\sqrt{T}}{P} \quad \text{or} \quad NDMF = \frac{W\sqrt{\frac{T}{288.15}}}{\frac{P}{101325}}$$

Equation 3

where W is the mass-flow, T is the total temperature, and P is the total pressure. Please note that Equation 3 is obtained under the assumption of ideal gas, but it is commonly used also for the performance representation of real components (compressor, turbines, etc.).

$$PR = \frac{P_{in}}{P_{out}}$$

Equation 4

where P_{in} is the pressure at the inlet of the component (in this case of the turbine), and P_{out} is the pressure at the outlet of the component.

In the following section, two cases are analysed:

1. simplified turbojet engine without reheat between the two turbine stages (NO ITR);
2. simplified turbojet engine with reheat between turbine stages (ITR).

CASE 1: NO ITR – First, the standard case without reheat between the turbine stages is discussed in order to demonstrate the basic concept of turbine operating point determination from stacked turbine maps. The concatenated turbine and nozzle maps are shown in Figure 2. The DP operating conditions' determination in the expansion section starts from the nozzle. Since the nozzle is choked, its map determines the maximum NDMF at its inlet (point (8) in Figure 2). Subsequently, the operating points of HPT stage 2 (HPT2) and HPT stage 1 (HPT1) can be determined by moving upstream: the NDMF at the HPT2 outlet (7) must equal the NDMF at (8). Thus, the characteristic curves of HPT2 determine the NDMF at the HPT2 inlet (6), which equals the NDMF at the HPT1 outlet (5). As long as the nozzle is choked, i.e. the operating point at (8) is on the straight section of the curve, the expansion ratio of each turbine stage is fixed.

Figure 2 - Expansion section maps without reheat

CASE 2: ITR – In the case reheat is introduced between HPT1 and HPT2, this causes a total temperature increase from station (5) to (6) as it is shown in Figure 3. Thereby, reheat alters the NDMF conservation. The relation between the NDMF at HPT1 outlet and HPT2 inlet is:

$$\frac{W_6\sqrt{T_6}}{P_6} = \frac{W_5\sqrt{T_5}}{P_5} \sqrt{\frac{T_6}{T_5} \frac{W_6}{W_5} \frac{P_5}{P_6}}$$

Equation 5

Given the assumptions listed previously, the latter two factors can be neglected and Equation 5 becomes:

$$\left(\frac{W\sqrt{T}}{P}\right)_6 = \left(\frac{W\sqrt{T}}{P}\right)_5 \sqrt{\frac{T_6}{T_5}}$$

Equation 6

The ratio $\sqrt{\frac{T_6}{T_5}}$ is purely a function of the operation of the second burner and shall therefore be referred to as Second Burner Temperature Ratio (SBTR):

$$SBTR = \sqrt{\frac{T_6}{T_5}}$$

Equation 7

Starting from a switched-off second burner (i.e. SBTR = 1), the progressive addition of heat in the reheat combustor causes an increase in the SBTR and in the difference between the NDMF at points (5) and (6). This is graphically represented by the vertical distance between the dashed red lines in Figure 3. Because the nozzle is choked, the NDMF at (6) is determined by the HPT2 and nozzle characteristics. Therefore, an increase in SBTR cannot increase the NDMF at (6) and will affect upstream components. In other words, the HPT1 outlet NDMF has to decrease to satisfy Equation 6. It is worth noting that all the aforementioned cases are definitions of different DP for a given engine configuration, and not the OD operation of a fixed engine.

Figure 3 - Expansion section maps with reheat

To summarise, it is evident that for engines with ITR, or multiple turbines coupled to the same shaft, the TWS has to be considered as an additional design parameter. Reheat alters the NDMF continuity between the outlet of turbine stage 1 and the inlet of turbine stage 2. With the given assumptions, and considering the design process outlined before, the operation of an ITR burner affects exclusively upstream turbine stages.

2.2. OD TURBINE THERMODYNAMICS

Further to the DP selection procedure described in section 2.1, this section describes the OD ‘matching’ of subsequent turbine sections, both without and with inter-stage reheat. The numerical determination of the new steady-state conditions is also presented. It shall be noted that the ITR engine offers an additional degree of freedom in engine control through the reheat outlet temperature of the second burner. In this context, the terms TET1 and TET2 will be used, referring to the first burner and reheat burner total outlet temperatures respectively.

‘Matching’ is defined as the process of readjustment and self-adaptation of all engine components, as a response to a change in operating conditions. This change can result from a variation in the inlet conditions, as well as from a purposeful throttling up or down by the control system through a change in the fuel flow(s). All the components in a gas turbine are connected thermodynamically through the gas path; moreover, certain components are mechanically connected by shafts. As a result of any external perturbation, the compressors, turbines and nozzles change their operating points according to their respective characteristics, until outlet and inlet NDMFs of each couple of subsequent components match, and work balance in each shaft is achieved. In this context, reheat alters the temperature between two subsequent turbine stages on the same spool and hence, directly influences their operating points. Consequently, the work split between turbine stages, which is imposed at DP by the turbine design, may change according to the turbine characteristics. The following sections assess under which conditions this may occur.

CASE 1: NO ITR – The standard case without reheat is considered first. As it was explained in the previous section, at DP conditions the operating points of HPT1 and HPT2 are determined by the nozzle choking NDMF. As long as the nozzle remains choked at OD, the operating points of the turbine stages are fixed.

Given the assumption of calorically perfect gas, even when the TET is changed, the work split between the two stages remains constant, as long as the nozzle is choked. In fact, the work produced by one turbine stage is:

$$TW = Wc_p(T_{in} - T_{out}) = Wc_pT_{in} \left(1 - \frac{T_{out}}{T_{in}}\right) \quad \text{Equation 8}$$

$$\frac{TW_{stage\ 1}}{TW_{stage\ 2}} = \frac{(T_{in1} - T_{out1})}{(T_{in2} - T_{out2})} = \frac{T_{in1}}{T_{in2}} \left(\frac{1 - \frac{T_{out1}}{T_{in1}}}{1 - \frac{T_{out2}}{T_{in2}}} \right) \quad \text{Equation 9}$$

Since $T_{out1} = T_{in2}$ (OD calculation but no reheat):

$$\frac{TW_{stage\ 1}}{TW_{stage\ 2}} = \frac{\frac{T_{in1}}{T_{out1}} - 1}{1 - \frac{T_{out2}}{T_{in2}}} = \frac{PR_1^{\eta_p \frac{\gamma-1}{\gamma}} - 1}{1 - PR_2^{-\eta_p \frac{\gamma-1}{\gamma}}} = const. \quad \text{Equation 10}$$

where γ is the heat capacity ratio, and η_p is the polytropic efficiency of the turbines. Since PR_1 and PR_2 are fixed, as long as the nozzle is choked, the turbine work split (TWS) remains fixed.

CASE 2: ITR – For ITR engines, two different cases are discussed here. In the first case, it is assumed that during any change of the turbine inlet temperatures, the SBTR remains constant at any OD step. Therefore, the vertical distance between points (5) and (6) in Figure 3 is fixed and the combined turbine-nozzle system behaves like a conventional expansion section as discussed previously. A change in TET1 will only affect the nozzle PR (point (8) in Figure 3 is free to move on the horizontal choking line, with constant inlet NDMF and variable PR) and the operating points of both turbine stages will still be constrained by the NDMF at station (8). As it was discussed in the previous “CASE 1”, the TWS remains fixed under these conditions.

In the second case analysed, SBTR is assumed to change due to the actions of the control system. A change in this ratio may occur for instance if TET1 (T_4) is kept constant while the TET2 (T_6) is increased. Hence, the SBTR (represented in Figure 3 by the vertical distance between points (5) and (6)) increases. Keeping in mind that the NDMF in (6) is fixed by the operating conditions of the downstream components, the increase in SBTR translates in HPT1 outlet NDMF (5) reduction. Therefore, the turbine characteristics enforce a reduction in HPT1 expansion ratio and hence, for a fixed TET1, the work produced by the HPT1 is reduced, as indicated by the equation for the turbine power:

$$TW = Wc_pT_{in} \left(1 - PR^{-\frac{\eta_p(\gamma-1)}{\gamma}}\right) \quad \text{Equation 11}$$

On the other hand, the NDMF at station (6) remains constant because the nozzle is choked. Therefore, the HPT2 expansion ratio remains constant. In combination with an increased inlet temperature, the work produced by the HPT2 is increased (see Equation 11). In conclusion, an increase in SBTR, apart from throttling up the engine, shifts the work split between the turbine stages to a higher loading of the HPT2.

To summarise, it has been demonstrated that TET1 does not affect work split, as long as TET2 is modulated in order to keep the SBTR constant. Conversely, changes in TET2, with constant TET1, will alter the work distribution between the stages (since SBTR changes). The new turbine work distribution between the stages depends on the individual component characteristics and must be determined by means of a numerical simulation. Because of the non-linearity of the components' characteristics and due to the simultaneous change of compressor pressure ratio and engine mass flow, an iterative approach must be used to solve the matching problem described above.

2.3. NUMERICAL CALCULATION OF OFF-DESIGN CONDITIONS

TURBOMATCH is a software developed by the Propulsion Engineering Centre at Cranfield University, and is extensively used by academia and industry in gas turbine performance simulation research. It comprises several pre-programmed modules, known as “bricks”. Most bricks correspond to models of individual gas turbine components, such as –but not exclusively– compressors, burners, turbines, mixers, nozzles, heat exchangers, flow splitters and power turbines. The program has been proven reliable, accurate, and extremely flexible [9] [10] [11] [12] [13].

The DP simulation in TURBOMATCH is a straightforward calculation that follows the components’ sequence in the main gas path. Since the user specifies all component parameters at DP, no iteration is required. In addition to this, during the DP calculation the components’ maps are scaled to match the DP specifications in terms of mass flow, PR, efficiency, etc. At OD, the determination of the new steady-state conditions requires an iterative approach because of the non-linearity of the components’ characteristics. TURBOMATCH makes use of the Newton-Raphson algorithm, which is an iterative solver for systems of non-linear equations.

The following paragraphs focus on the general approach for OD calculations with ITR, while the mathematical aspects of the solver are not discussed in this document. The explanation makes use of terms that are defined beforehand:

- **Handles** are free parameters that are set by the user to control engine operation. Typically, the combustor outlet temperature (in the hereafter illustrated example and validation case, this coincides with the turbine entry temperature), the fuel flow, or the compressor relative rotational speed (PCN) –as defined in Equation 12– is employed as a numerical handling parameter. The manipulation of the handle mimics the function of the control system of a real engine.

$$PCN = \frac{N}{N_{DP}}$$

Equation 12

- **Guesses** are variables that are modified internally by the algorithm to find the new equilibrium point according to the given handles. Therefore, a guess is a degree of freedom (i.e. a variable) for the iterative solver.
- **Checks** are mechanical and thermodynamic constraints that have to be satisfied. The program internally expresses them as errors and modifies the guesses, until all errors are zeroed simultaneously. This identified set of ‘guessed’ values represents the new equilibrium point of the engine. In order to be able to find one and only one solution, it is important that the number of checks equals the number of guesses.

The process of determining the handles, guesses and checks within an iterative OD calculation for an ITR engine will be illustrated hereafter using the simplified turbojet model as shown in Figure 1. The DP simulation of an engine with ITR, compared to a non-ITR calculation, only requires the additional specification of the work split between the turbine sections on the same shaft. In the following discussion, the DP calculation is assumed to have been completed previously, therefore all component maps are already scaled accordingly.

1. Compressor The algorithm starts with the guesses for the compressor pressure ratio (CPR –*first guess*) and the PCN (*second guess*). As it is shown on the illustrative compressor map in Figure 4 [9], these two guesses combined univocally determine an operating point. Therefore, the engine’s mass flow and the compressor isentropic efficiency are extracted from the map and the power consumed by the compressor is calculated.

Figure 4 - Specification of the operating point with two guesses

2. First burner The outlet temperature of the first burner is the *first handle* and therefore, is specified by the user. Since the combustor inlet conditions are known from the compressor calculation, the fuel flow in the burner can be calculated.

3. Turbine stage 1 With the PCN (*second guess*) and the TET1 (*first handle*), the non-dimensional rotational speed of the turbine can be calculated and the speed line in the turbine map can be defined. The *third guess* within the algorithm is the HPT1 PR. Subsequently, the turbine inlet NDMF and isentropic efficiency are read from the map. The information available allows calculating the turbine outlet pressure and temperature, together with the power produced by the first turbine stage.

At this point, the *first check* can be carried out to quantify the difference between the NDMF at the outlet of the first burner and at HPT1 inlet.

4. Second burner The second burner outlet temperature is the *second handle* defined by the user.

5. Turbine stage 2 The determination of the HPT2 speed line, NDMF and stage work is equivalent to the calculation completed for the first turbine stage. The *fourth guess* is the HPT2 PR. The difference in NDMF between burner 2 outlet and HPT2 inlet is the *second check*, which is a function of all four previous guesses. At this point of the calculation, the power balance between the compressor and the combined work of the turbine stages can be examined (*third check*).

6. Nozzle Knowing the HPT2 exit total pressure and the ambient pressure (or critical pressure in the case of a choked simple convergent nozzle), the nozzle PR is calculated. The NDMF at the nozzle inlet, which is obtained from the nozzle map, is compared to the NDMF at the HPT2 outlet (*fourth check*).

In conclusion, the second combustor provides an additional opportunity for engine control through the reheat outlet temperature. An iterative process to calculate the OD conditions of an ITR gas generator must consider both temperatures as handles. The algorithm is summarised in Annex A, where a flow chart for the process is provided together with a table summarising guesses and checks involved.

3. RESULTS AND DISCUSSION

3.1. METHODOLOGY VERIFICATION

After implementing the abovementioned methodology in TURBOMATCH, the code was verified to ensure robustness before applying it to any further engine studies. The software verification had to include performance comparisons across a range of part-speed operating points.

The only contemporary gas turbines with ITR are the ALSTOM GT24 and GT26, which are large-scale single-shaft gas turbines for combined-cycle power generation. The latter, which is the 50Hz version for the European market, has a gross electrical output of

345 MW and an exhaust mass flow of 715 kg/s. This engine features ITR between the first and second turbine stage and, therefore, belongs to the class of gas turbines for which the new version of TURBOMATCH was developed. Unfortunately, engine data available in the public domain was found to be insufficient for the setup of an accurate model of this gas turbine [16].

For the verification of the ITR functionality in TURBOMATCH, it was decided to carry out a comparison against GasTurb. GasTurb is a commercial, closed-source, non-modular performance code, where the user can select an engine set-up from a list of built-in templates. In terms of ITR, its modelling capability is limited to single-shaft turboshaft configurations. For the comparison, the simplest configuration was chosen, being the single-shaft turboshaft with ITR that is similar to the ALSTOM GT26. Compressor bleeds and turbine cooling flows were not considered for simplicity purposes and consistency between the two programs. Components' parameters were chosen arbitrarily (see Table 1) and were specified without any modifications in both software tools. GasTurb component maps were also used in TURBOMATCH, again on the grounds of consistency. For OD calculations, the TET1 was used as handle, while the ratio TET2/TET1 was kept fixed.

Handle: TET1 = 1600K			
mass flow	400 kg/s	π_{intake}	0.99
CPR	20	η_{compr}	0.85
TET	1600K	η_{burn}	1
PLI	4	π_{burn}	0.95
TET2/TET1	1	η_{turb}	0.9

Table 1 - Engine design point parameter for validation example

In Table 1, the Power Law Index (PLI) represents how the shaft output power relates to the shaft rotational speed, and is defined as follows:

$$Power = Power_{DP} \cdot PCN^{PLI} \quad \text{Equation 13}$$

The definition of PLI, as given in Equation 13, is convenient from a numerical point of view because it allows to solve the matching problem of an industrial gas turbine in the same way as described for a turbojet engine in section 2.3. In fact, known the shaft output power at DP and the PLI, the guessed PCN for the OD calculation allows to calculate the corresponding shaft output power.

A first comparison was made upon the DP performance (see TET=1600 K in Table 2). Generally, a high degree of conformity was observed; the differences between the two software in predicted power output, fuel flow, thermal efficiency, turbine work and exhaust gas temperature were lower than 2%.

During the comparison of pressures and temperatures, a discrepancy in the compressor delivery temperature of 0.5% was observed for DP operating conditions. More specifically, while the inlet temperature and DP PR of both compressors were the same, the temperature ratio across the compressor was different. The temperature ratio can be expressed as:

$$\frac{T_{out}}{T_{in}} = \left(\frac{P_{out}}{P_{in}} \right)^{\alpha} \quad \alpha = f(\eta_p, \text{gas properties}) \quad \text{Equation 14}$$

Hence, the observed deviation is ascribed to α and, therefore, to differences in the gas models used by TURBOMATCH [17] and GasTurb. This was also confirmed by previous studies comparing GasTurb with other performance codes [18], where the same conclusion was reached. Consequently, minor discrepancies were expected also in OD calculations, as no correction was introduced to address this minor issue.

Subsequently, the turbine entry temperature was reduced from 1600K to 1420K in steps of 20K, and relative differences in all main gas turbine performance parameters – power output, fuel flow, thermal efficiency, rotational speed, compressor pressure ratio and air mass flow – were analysed. The aforementioned differences between GasTurb and TURBOMATCH are shown in Table 2.

TET [K]	Power Out	Fuel Flow	Th. Efficiency	PCN	CPR	Mass Flow
1600	1.27%	1.60%	0.28%	0.00%	0.00%	0.00%
1580	1.90%	1.83%	0.06%	0.16%	0.42%	0.43%
1560	2.17%	1.90%	0.27%	0.23%	0.61%	0.62%
1540	2.04%	1.71%	0.34%	0.20%	0.49%	0.53%
1520	1.82%	1.43%	0.40%	0.14%	0.22%	0.33%
1500	1.65%	1.15%	0.51%	0.10%	0.05%	0.16%
1480	2.39%	1.39%	1.01%	0.29%	0.45%	0.61%
1460	2.80%	1.41%	1.41%	0.39%	0.67%	0.87%
1440	3.12%	1.40%	1.75%	0.48%	0.86%	1.09%
1420	3.48%	1.40%	2.10%	0.57%	1.08%	1.35%

Table 2 - Relative performance difference at off-design (GasTurb represents the baseline code; the colours' range, from green to red, provides a rapid overview of more consistent -green- and less consistent -red- parameters in the comparison)

In the investigated range, the OD differences were:

- lower than 1.5% for CPR, mass flow, PCN and fuel;
- lower than 3.5% for power output and thermal efficiency.

To further assess the similarities between the results, the running lines of the compressor, HPT1 and HPT2 were plotted together for a visual comparison. The results are shown in Annex B. All running lines were found to reasonably coincide between the two codes.

In the light of the negligible uncertainties and the high level of overall similarity between the two codes, it was concluded that the ITR modification in TURBOMATCH was successful and capable of producing reliable results. Inherent factors that may impact on the difference between both algorithms and which cannot be adjusted are:

- gas model – heat capacity at constant pressure (c_p) and γ as a function of total gas temperature differ in the two codes;
- different combustor maps.

To summarise, the proposed simulation technique was proven to be capable of modelling engine configurations with ITR. Therefore, it can be used in the analysis and optimisation of reheated industrial or aero engines, and in any other study where extensive and accurate OD modelling of ITR configurations is required.

3.2. TURBINE WORK SPLIT VARIATION AT OD

The proposed methodology for the simulation of multiple turbines on the same shaft offers different advantages depending on the specific requirements of the simulation task. For example, both aero engines and industrial gas turbines with ITR can be modelled and simulated more accurately by capturing the effects of multiple turbines on the same shaft or the effects of turbines' cooling flows arrangement, where the inter stage cooling can be modelled in a more detailed way. All this can have great impact not only on engine performance prediction, but also on gas-path diagnostic [19] and prognostic [20] methods, on real time engine simulations [21], and on the accuracy of thermodynamic cycles comparison and selection [22] [23].

In all the aforementioned cases, the proposed technique is advantageous by calculating the TWS, depending on the turbine characteristics and combustor operation. In this last paragraph, the change in TWS at OD is quantified for different configurations and different engine control approaches. This quantification shows the importance of implementing the methodology described in this paper, in order to get accurate results in ITR engine configurations.

Four different cases are considered when assessing the change in TWS during OD operation of a simple turbojet engine, as listed hereafter:

- Case 1: reheat not operational, no cooling flows;
- Case 2: reheat not operational, cooling flows layout as depicted in Figure 5. The model is introducing the following cooling flows: 10% of the main gas path air flow is extracted for HPT1 NGVs cooling, followed by 7% of main gas path air flow extraction for HPT1 rotor cooling, followed by 5% of main gas path air flow extraction for HPT2 NGVs cooling, followed by 3% of main gas path air flow extraction for HPT2 rotor cooling;
- Case 3: reheat operational, with $TET_2 = TET_1$;
- Case 4: reheat operational, with $TET_{2OD} = TET_{2DP}$;

Handle: TET1 = 1900K	
mass flow	100 kg/s
CPR	40
TWS	0.5

Table 3 - DP parameter for turbojet

Figure 5 - Cooling flow arrangement for Case 2

Table 3 shows the main DP parameters chosen for the turbojet model, while Figure 6 illustrates the values for the TWS of all four cases for TET1 ranging between 1900K (DP) and 1300K. According to the theoretical considerations from above (Equation 10), the TWS in Case 1 should remain constant since the SBTR is unity for every operating point. However, as seen in the diagram, the TWS increases slightly at OD, but remains below 0.51 within the investigated TET range. This change in TWS of less than 2% is ascribed both to the temperature dependency of the gas properties and to the realistic turbine maps implemented in TURBOMATCH, compared to the ideal gas model and simplified single speed line maps considered within the theoretical discussion.

In Case 2, cooling flows for the two turbine stages were considered, in order to investigate if the turbine inlet temperature alteration, as a consequence of the cooling flows' injection, impacts on the TWS at OD. It can be seen in the diagram that this impact is only marginal. In the present example, the TWS remains within a range between 0.5 and 0.503 and therefore closer to the DP value than in the case of the uncooled turbojet. The same considerations described for Case 1 apply.

In Case 3, the reheat burner is operational and therefore, it increases the NDMF HPT1 outlet and HPT2 inlet. However, with the operational choice of TET2=TET1 the SBTR change is marginal at OD. As largely described in the theoretical discussion on the OD thermodynamics of ITR, the TWS remains constant under these circumstances. This behaviour is supported by the data in Figure 6, where the respective trend line coincides with the data from the first case, where the SBTR was kept constant, equal to unity.

However, in Case 4 the SBTR is subjected to a significant variation, because of the gradual TET1 decrease in combination with a constant TET2. Hence, T_5 decreases and the SBTR increases (Equation 7). As it was observed in the theoretical considerations (see section "OD Turbine Thermodynamics - Case 2"), this causes a drop in the TWS. The predicted trend is confirmed by the green line plotted in Figure 6. Consequently, it can be concluded that a simulation methodology, which allows the TWS between separate turbine stages to vary, is of particular importance for cases with changes in the SBTR (or, similarly, in the TET2/TET1 ratio).

Figure 6 - TWS at OD for the four example cases

The results presented above were produced with generic maps for the compressor and turbines. More specifically, the results in Figure 6 were produced by using the same map in both turbine sections, although they were scaled differently, according to turbines' DP requirements. In addition to this, further simulations were carried out with different maps, showing that the trend for the TWS at OD is sensitive to the shape of the maps implemented.

4. CONCLUSIONS

This paper presented a methodology for the OD simulation of gas turbine engines with ITR. The procedure of OD matching for conventional and ITR configurations was illustrated using a simple turbojet engine as an example.

Based on first-principles analysis, a numerical technique for the determination of OD steady-state conditions was developed and integrated into TURBOMATCH. The code was compared against a commercial performance simulation software, using a simple turbojet engine with ITR. OD operating conditions were defined by progressively throttling down the engine, with a maximum reduction of 180 K in TET1 and TET2. The results show differences lower than 3.5% for power output and thermal efficiency, and lower than 1.5% for all other relevant thermodynamic parameters. These discrepancies are mainly ascribed to the different gas models implemented in the codes compared. In addition, it was shown that for specific OD conditions where TET1 varied from 1900 to 1300 K with constant TET2, the TWS dropped by almost 30% compared to DP.

In general, the presented simulation technique is essential for the performance assessment of engines with ITR. Furthermore, it enables a detailed modelling of complex turbine cooling arrangements with multiple cooling flows and reinjection points within the turbine expansion section, as it is typically the case in multi-stage single-shaft industrial gas turbines.

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ANNEX A – GUESSES AND CHECKS FOR AN OD CALCULATION OF A TURBOJET WITH ITR

Handle: T_4, T_6

Guesses:

1. CPR (2) $\rightarrow$ (3)
2. Rotational speed PCN
3. PR HPT1 (4) $\rightarrow$ (5)
4. PR HPT2 (6) $\rightarrow$ (7)

Checks:

1. NDMF at combustor outlet (4) = NDMF at HPT1 inlet
2. NDMF at reheat outlet (6) = NDMF at HPT2 inlet
3. NDMF at HPT2 outlet (8) = NDMF at nozzle inlet
4. Work balance

Table 4 - Guesses and Checks for an OD calculation of a turbojet with ITR

Figure 7 - Flow chart for an OD calculation of a turbojet with ITR. Numbers on arrows represent the sequence of inputs/outputs at each step of the calculation

ANNEX B – RUNNING LINE COMPARISON FOR VALIDATION

Figure 8 - GasTurb vs TURBOMATCH – Compressor map

Figure 9 - GasTurb vs TURBOMATCH – Turbine 1 map

Figure 10 - GasTurb vs TURBOMATCH - Turbine 2 map

Highlights

- An innovative gas turbine performance simulation methodology is proposed
- It allows to perform DP and OD performance calculations for complex engines layouts
- It is essential for inter-turbine reheat (ITR) engine performance calculation
- A detailed description is provided for fast and flexible implementation
- The methodology is successfully verified against a commercial closed-source software

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